CORRIGE M A T H E M A T I Q U E S G 1^{er} GROUPE 2015

15 T 37 A 01

EXERCICE 1 **(05 points)**

1) Soit la matrice A = $\begin{pmatrix} 1 & 2 & -1 \\ 2 & -1 & 5 \\ 1 & 2 & -3 \end{pmatrix}$; $|A|$ = 10

$$\begin{matrix} L_1 \\ L_2 \\ L_3 \end{matrix} \left(\begin{matrix} 1 & 2 & -1 \\ 2 & -1 & 5 \\ 1 & 2 & -3 \end{matrix} \quad \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\right) \Rightarrow \begin{matrix} L_1 = L_1 \\ L_2 = L_2 - 24 \\ L_3 = L_3 - 4 \end{matrix} \left(\begin{matrix} 1 & 2 & -1 \\ 0 & -5 & 7 \\ 0 & 0 & -2 \end{matrix} \quad \begin{matrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{matrix}\right) \Rightarrow A^{-1} = \begin{pmatrix} \frac{-7}{10} & \frac{2}{5} & \frac{9}{10} \\ \frac{11}{10} & \frac{-1}{5} & \frac{-7}{10} \\ \frac{1}{2} & 0 & \frac{-1}{2} \end{pmatrix}$$

$$\begin{cases} x + 2y - z = 2 \\ 2x - y + 5z = 6 \\ x + 2y - 3z = 0 \end{cases} \Rightarrow AX = B, \quad A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & -1 & 5 \\ 1 & 2 & -3 \end{pmatrix}, X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, B = \begin{pmatrix} 2 \\ 6 \\ 0 \end{pmatrix}$$

AX = B $\Rightarrow$ X = A^{-1}B $\Rightarrow$ X = $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ donc S = $\{(1,1,1)\}$.

EXERCICE 1

2) $\begin{cases} x \geq 0, y \geq 0 \\ 6x + 2y \leq 36 \\ 5x + 5y \leq 40 \\ 2x + 4y \leq 28 \\ f = 5x + 3y \end{cases} \Rightarrow \begin{cases} x \geq 0, y \geq 0, t_1, t_2, t_3 \geq 0, \\ 6x + 2y + t_1 = 36 \\ 5x + 5y + t_2 = 40 \\ 2x + 4yt_3 = 28 \\ f = 5x + 3y \end{cases}$

	x	y	t_1	t_2	t_3	C
L_1 t_1	6	2	1	0	0	36
L_2 t_2	5	5	0	1	0	040
L_3 t_3	2	4	0	0	1	28
L_4 f	5	3	0	0	0	0

	x	y	t_1	t_2	t_3	C
L'_1 =1/6 L_1 x	1	1/3	1/6	0	0	6
$L'_2 = L1 - 5L'_1$ e2	0	10/3	-5/6	1	0	10
$L'_3 = \mathrm{L3} - 2L'_1$ e3	0	10/3	-1/3	0	1	16
L'_4=$L'_4 - L_1$ x	0	4/3	-5/6	0	0	-30

………..x = 5, y = 3 et f = 36

PROBLEME

f(x) = (x^2 – 3)e^{-x}

Df = IR

$\lim\limits_{x \to -\infty} f(x) = +\infty$ $\lim\limits_{x \to +\infty} f(x) = 0 \Rightarrow$ y = 0 AH

$\lim\limits_{x \to -\infty} \frac{f(x)}{x} = -\infty \Rightarrow$ (C) admet une branche parabolique en , de direction (oy).

f'(x) = (– x^2+2x+3)e^{-x} d'où f'(x) = 0 $\Rightarrow$ S $\{-1,3\}$.

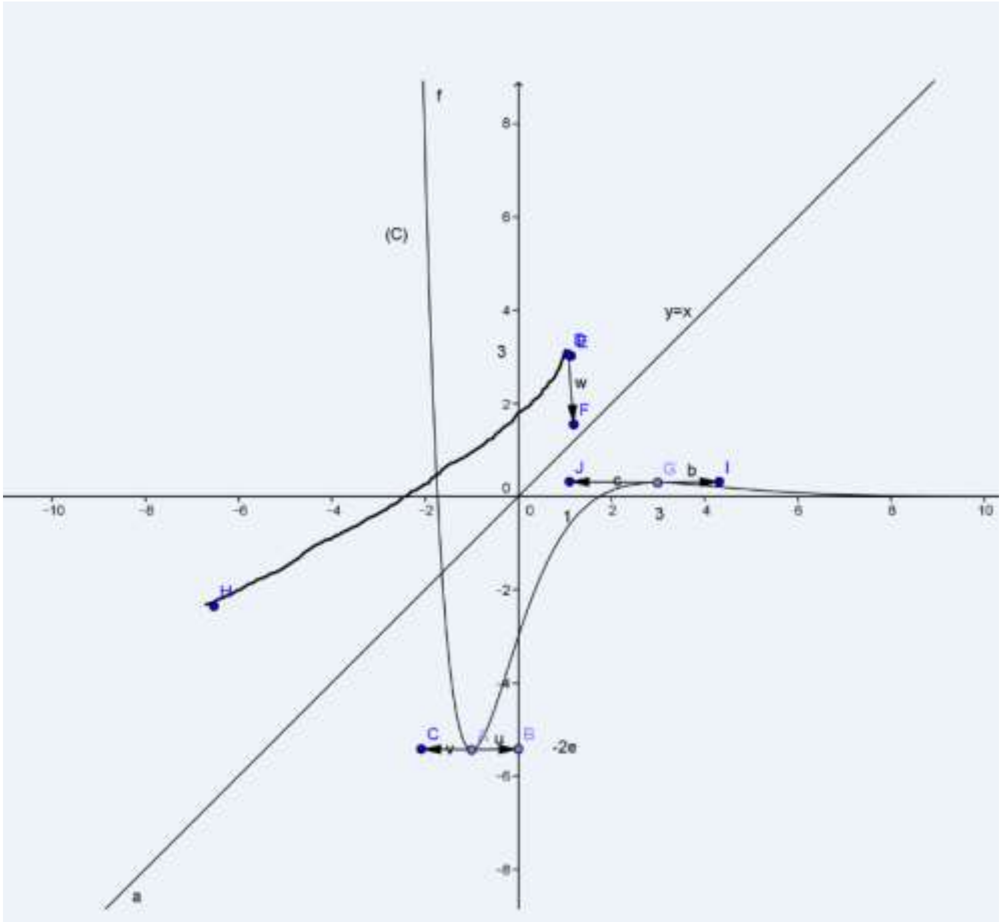
x	$-\infty$		-1		3		$+\infty$
f'		$-$	0	$+$	0	$-$	
$f(x)$	$+\infty$	$\searrow$	$-2e$	$\nearrow$	$6e^{-3}$	$\searrow$	0

f(x) = 0 ⇒ x= $-\sqrt{3}$ ou $\sqrt{3}$ ⇒(C)∩(0x) = $\{A(-\sqrt{3}, 0), B\ (\sqrt{3}, 0)\}$

E(0,f(o) = (0,−3) ⇒ y = 3x-3.

F est continue et croissante sur $[-1{,}3]$ donc f est une bijection J = $[-1{,}3]$ sur K = $[-2e, 6e^{-3}]$.

$(f^{-1})'(-3) = \frac{1}{f'(f^{-1}(-3))}$) = $\frac{1}{f'(0)}$) = $\frac{1}{3}$ *alors f(o) =−3.*



f(x) = m

si m $\in]-\infty, -2e[$*, on a 0 solution.*

si m $\in]-2{,}0[$*, on a 2 solutions.*

si m $\in]0{,}6e^{-3}[$*, on a 3 solutions.*

si m $\in]6e^{-3} + \infty[$*, on a 1 solution.*

si m = $-2e$*, on a 1 solution.*

F(x) = (ax^2+ bx + c) e^{-x}.

F'(x) = f(x) ⇔ $\begin{cases} a = 1 \\ b = -2 \\ c = 1 \end{cases}$

F(x) = (ax^2- 2x + 1) e^{-x}.

A(α) = $\int_3^{\alpha} f(x)dx = \left[(-x^2-2x+1)e^{-x}\right]_3^{\alpha} = (-\alpha^2-2\alpha+1)e^{-\alpha}$ *+1*+e^{-3}

⇒ A(α) *est l'axe du domaine limite sur C, l'axe des abscisses et la droite d'abscisse x = 3, x =* α

$\lim_{x \to +\infty} f(x) =$ *14e^{-3}*